

EFFECTS OF EXCHANGE RATE IN LEARNING INFLATION

Research Assistant. Aytug Zekeriya BOLCAN¹

Reference: Bolcan, A.Z. (2020). "Effects Of Exchange Rate In Learning Inflation", International Journal of Disciplines Economics & Administrative Sciences Studies, Vol:6, Issue:23; pp:712-715

ABSTRACT

The objective of this primary study is to investigate the effects of real exchange rate on inflation with learning mechanism in floating exchange rate rule. More specifically we look at high inflation and high government deficit period. Marcet and Nicolini (2003) Model is used to approach this issue. But we use Floating Exchange Rate Rule, rather than Fixed Exchange Rate Rule. For simplicity, we also assume balanced trade.

Keywords: Learning Models; Inflation; Exchange Rate Rules

1. MODEL

In our model Money Demand will be equal to

$$\frac{M_t}{P_t} = -\alpha_1 E_t \left[\frac{P_{t+1}}{P_t} \right] - \alpha_2 \epsilon_t$$

where $E_t \left[\frac{P_{t+1}}{P_t} \right]$ is expected inflation, ϵ_t is nominal exchange rate, α_1 is elasticity of money demand to expectation of inflation and α_2 is elasticity of money demand to nominal exchange rate.

Agents are less willing to hold money in high inflation, so we will expect α_1 will be negative ($-\alpha_1$). Also, if Nominal exchange rate increase, agents want to hold foreign currency, then they will sell domestic currency to buy foreign currency α_1 is negative too ($-\alpha_1$).

On the other hand, Nominal Exchange Rate equal to

$$\epsilon_t = e_t + E_t \left[\frac{P_{t+1}}{P_t} \right]$$


where e_t is Real Exchange Rate. Take Equation 2 and use in Equation 1 and rearrange it;

$$\begin{aligned} M_t &= -\alpha_1 E [P_{t+1}] - \alpha_2 P_t \epsilon_t \\ -\alpha_2 P_t \epsilon_t &= M_t + \alpha_1 E [P_{t+1}] \\ P_t (-\alpha_2 \epsilon_t) &= \alpha_2 P_t e_t \alpha_2 P_t \frac{E [P_{t+1}]}{P_t} \\ &= P_t (-\alpha_2 e_t) - \alpha_2 E [P_{t+1}] \\ P_t (-\alpha_2 e_t) &= M_t \alpha_1 E [P_{t+1}] + \alpha_2 E [P_{t+1}] \\ P_t (-\alpha_2 e_t) &= M_t + (\alpha_1 + \alpha_2) E [P_{t+1}] \\ P_t &= \frac{1}{-\alpha_2} \frac{1}{e_t} M_t + \frac{\alpha_1 + \alpha_2}{-\alpha_2} \frac{1}{e_t} E [P_{t+1}] \end{aligned}$$

If $\frac{1}{\alpha_2} = \gamma$ and $\frac{\alpha_1 + \alpha_2}{-\alpha_2} = \delta$ then

$$P_{\{t\}} = \delta \frac{1}{e_t} E [P_{t+1}] + \gamma \frac{1}{e_t} M_t$$

¹ Research Assistant in Department of Economics, Faculty of Economics, Istanbul University, İstanbul/Turkey

 <https://orcid.org/0000-0002-5162-4337>

Government Budget Constraint is

$$\frac{M_t - M_{t-1}}{P_t} - \frac{R_t - R_{t-1}}{P_t} e_t = d$$

where M_t is money supply, d_t is Seigniorage. The government does not tax agents. Money is not exogenous and is actually set by a central authority to raise seigniorage income that finance government deficit.

Money in the economy, whether it is cash or whether it is bank deposits in the central bank, is something that Central Bank creates with has zero cost. Central Bank also raises revenue by changing its stock of foreign currency R_t . Lastly, $\frac{R_t - R_{t-1} P_t}{e_t}$ equal to Trade Balance.

We rearrange last equation and write

$$\begin{aligned} \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} - \frac{R_t}{P_t} + \frac{R_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} e_t &= d \\ \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{1}{\pi_t} - \frac{R_t}{P_t} + \frac{R_{t-1}}{P_{t-1}} \frac{1}{\pi_t} e_t &= d \end{aligned}$$

If Trade is balanced then $\frac{R_t - R_{t-1}}{P_t} e_t = 0$ and budget constraint will be

$$\begin{aligned} \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} &= d_t \\ \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{1}{\pi_t} &= d_t \end{aligned}$$

Here we also assume that real exchange rate is determine by endogenously ($e_t = e$).

How does this model behave under Rational Expectation? Under Rational Expectation, we can expect that $E(P_{t+1}) = P_{t+1}$, because there is no uncertainty. We rewrite Equation $P_{\{t\}} = \delta \frac{1}{e_t} E [P_{t+1}] + \gamma \frac{1}{e_t} M_t$,

$$\begin{aligned} 1 &= \delta \frac{1}{e} \frac{P_{t+1}}{P_t} + \gamma \frac{1}{e} \frac{M_t}{P_t} \\ \frac{M_t}{P_t} &= \left(1 - \delta \frac{1}{e} \frac{P_{t+1}}{P_t} \right) \frac{1}{\gamma} e \end{aligned}$$

When we use Equation $\frac{M_t}{P_t} = \left(1 - \delta \frac{1}{e} \frac{P_{t+1}}{P_t} \right) \frac{1}{\gamma} e$ in Equation $\frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} = d_t$, we will have

$$\left(1 - \delta \frac{1}{e} \frac{P_{t+1}}{P_t} \right) \frac{1}{\gamma} e - \left(1 - \delta \frac{1}{e} \frac{P_t}{P_{t-1}} \right) \frac{1}{\gamma} e \frac{P_{t-1}}{P_t} = d_t$$

$$\left(1 - \delta \frac{1}{e} \frac{P_{t+1}}{P_t} \right) \frac{1}{\gamma} e - \left(e \frac{P_{t-1}}{P_t} - \delta \right) \frac{1}{\gamma} = d_t$$

$$e - \delta \pi_{t+1} - \left(e \frac{1}{\pi_t} - \delta \right) = \gamma d_t$$

$$-e + \delta \pi_{t+1} + \left(e \frac{1}{\pi_t} + \delta \right) = -\gamma d_t$$

$$\delta \pi_{t+1} = -\gamma d_t + e - \left(e \frac{1}{\pi_t} + \delta \right)$$

$$\pi_{t+1} = -\frac{1}{\delta}\gamma d_t + \frac{1}{\delta}e\left(1 - \frac{1}{\pi_t}\right) + 1$$

we could introduce learning like agents have expected inflation (\tilde{E}), we have to write agents model,

$$P_t = \delta \frac{1}{e} \tilde{E}[P_{t+1}] + \gamma \frac{1}{e} M_t$$

we assume constant shock.

$$\pi_t = \beta + u_t$$

We assume that the learning mechanism is given by the stochastic approximation algorithm

$$\beta_t = \beta_{t-1} + \frac{1}{t}(\pi_{t-1} - \beta_{t-1})$$

Constant gain will be optimal, if you want to give more weight to the last observations like our interest, high inflation period.

$$P_t = \delta \frac{1}{e} \beta_t P_t + \gamma \frac{1}{e} M_t$$

$$P_t - \delta \frac{1}{e} \beta_t P_t = \gamma \frac{1}{e} M_t$$

$$\left(1 - \delta \frac{1}{e} \beta_t\right) P_t = \gamma \frac{1}{e} M_t$$

$$\left(1 - \delta \frac{1}{e} \beta_t\right) \frac{e}{\gamma} = \frac{M_t}{P_t}$$

$$\frac{e}{\gamma} - \frac{\delta}{\gamma} \beta_t = \frac{M_t}{P_t}$$

When we use Equation $\frac{e}{\gamma} - \frac{\delta}{\gamma} \beta_t = \frac{M_t}{P_t}$ in Equation $\frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} = d_t$, we will have

$$\frac{e}{\gamma} - \frac{\delta}{\gamma} \beta_t - \left(\frac{e}{\gamma} - \frac{\delta}{\gamma} \beta_{t-1}\right) \pi_t^{-1} = d$$

$$e - \delta \beta_t - (e - \delta \beta_{t-1}) \pi_t^{-1} = \gamma d$$

$$e - \delta \beta_t - \gamma d = (e - \delta \beta_{t-1}) \pi_t^{-1}$$

$$\pi_t = \frac{e - \delta \beta_{t-1}}{e - \delta \beta_t - \gamma d}$$

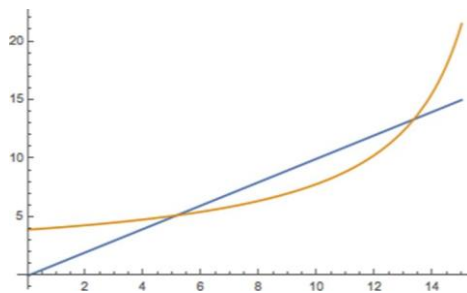
In high inflation period, it is more likely, the forecasting error is high. So we are using constant gain. Constant gain is optimal you want to give more weight to the last observations Williams (2018). Then for large t , β_{t-1} will be close to β_t .

$$\pi_t = \frac{e - \delta \beta_t}{e - \delta \beta_t - \gamma d}$$

2. DYNAMICS OF LEARNING

We need to make assumption on α_1 and α_2 . We assume that α_2 is high which means that agents are very sensitive to changes in exchange rate and also to changes Prices. What does Equation $\pi_t = \frac{e - \delta \beta_t}{e - \delta \beta_t - \gamma d}$ tell us?

$$\{e = 55, d = 41, \delta = 0.8, \gamma = 1.\}$$



$$\{e = 55, d = 43, \delta = 0.8, \gamma = 1.\}$$

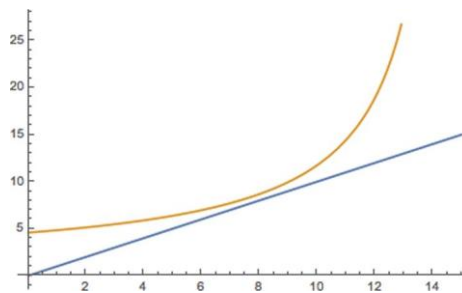
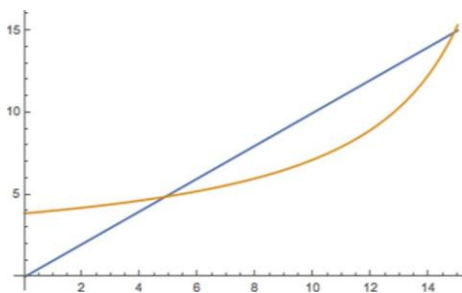


Figure 1 . An Increase in Government Deficit

As we see in Figure 1, when constant exchange rate, even if a slight increase in the deficit, increases the probability of high and then hyper-inflation. This result also supports the results of Marcet and Nicolini (2003).

On the other hand, an increase in reel exchange rate (decrease of value of domestic currency), compensate adverse effect of deficit, as we see in Figure 2

$$\{e = 58, d = 43, \delta = 0.8, \gamma = 1.\}$$



$$\{e = 60, d = 43, \delta = 0.8, \gamma = 1.\}$$

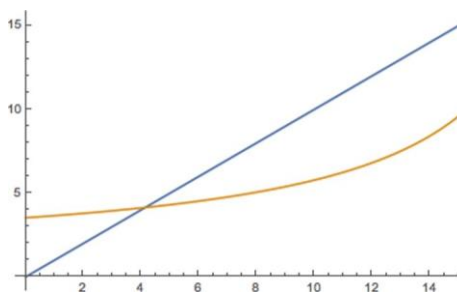


Figure 2. An Increase in Exchange Rate

3. CONCLUSION

In this study, we try to explain how agents reacts changes in exchange rate while they are learning to the inflation rate.

In further studies, we will first calibrate elasticities of money demand to expectation of inflation and, to nominal exchange rate, then we search unbalanced trade, which is one of the major problems in emerging countries, to elaborate your results.

REFERENCES

Marcet, Albert, and Juan P Nicolini. 2003. "Recurrent and Learning Hyperinflations." *The American Economic Review*, 93(5): 1476-1498.

Williams, Noah. 2018. "Escape Dynamics in Learning Models." *The Review of Economic Studies*, 86(2): 882{912.